# Mathematical Fuzzy Logic

**Ph.D. course**

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May – June 2009

**Aims of the course:** To introduce the basic notions of Mathematical Fuzzy Logic and some of the open problems in its agenda.

**Topics:**

1. Introduction to the algebraic study of propositional logics. The cases of intuitionistic and classical logic. The general notion of algebraizable logic.

2. Gödel-Dummett logic as an extension of intuitionistic logic. Product and Łukasiewicz logic. Algebraic semantics. Standard semantics given by continuous t-norms. Finite strong standard completeness properties and failure of strong standard completeness.

3. Hájek's BL logic as the logic of all continuous t-norms and their residua. Algebraic semantics, ordinal sums and standard completeness properties. Hilbert style calculus.

4. Residuation and left-continuity. MTL as the logic of all left-continuous t-norms and their residua. Algebraic semantics, Hilbert style calculus and Local Deduction Theorem. Real embedding property.

5. Core fuzzy logics. Completeness properties with respect to distinguished semantics (real, rational, hyperreal and finite chains): methods and equivalencies.

6. First-order predicate fuzzy logics: axiomatization and semantics. Completeness properties.

7. Modal fuzzy logics.

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